УПРУГО И УПРУГО-ПЛАСТИЧЕСКОЕ ДЕФОРМИРОВАНИЕ ВОЛОКОН ПРИ ОСЕВОМ НАГРУЖЕНИИ ПРЯЖИ

ELASTIC AND ELASTIC-PLASTIC DEFORMATION OF FIBERS UNDER AXIAL LOADING IN TWISTED YARN

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РЕФЕРАТ

УПРУГАЯ И УПРУГО-ПЛАСТИЧЕСКАЯ ДЕФОРМАЦИЯ, ДЕФОРМАЦИОННОЕ СОСТОЯНИЕ ВОЛОКНА, УДЛИНЕНИЕ, ОСЕВОЕ НАПРЯЖЕНИЕ

Предметом исследований являются законы деформирования в осевом направлении текстильных нитей. Реальная нить представлена в виде совокупности большого числа элементов, обладающих простейшими законами деформирования с разными константами. Рассмотрен случай, когда волокна в зависимости от осевого нагружения и кручения пряжи деформируются по упругому или упруго-пластическому законам по схеме Прандтля. Установлено, что с ростом угла кручения волокна в сечениях пряжи больше растягиваются и поэтому их пластическое деформирование происходит при более высоких значениях осевой деформации. Разработана теоретическая модель деформирования волокон в зависимости от величин осевого усилия и угла крутки пряжи. Для описания развития пластической зоны деформирования волокон предложено использовать схему Прандтля, согласно которой связь между деформацией волокон и осевым напряжением представляется двумя прямыми линиями. Результаты работы могут быть использованы при прогнозировании и оценке показателей механических свойств текстильных нитей.

ABSTRACT

ELASTIC AND ELASTIC-PLASTIC DEFORMATION, STRAIN STATE OF FIBER, ELONGATION, AXIAL STRESS

The paper discusses developed of theoretical model of deformation of the fibers, depending on the values of axial force and the twist angle of the yarn. To describe the development of plastic fibers deformation zone it is proposed to use the Prandtl scheme, according to which the connection between the fiber strain and axial stress is represented by two straight lines (bilinear diagram). The condition of axial strain is found when all fibers in the yarn are in a state of plastic deformation. In the absence of hardening the tension in the plastic zone for the twist angle between some values may decrease up to 1.5–3 times. Thus, with the presence in the yarn fibers deformable by elastically plastic according Prandtl law scheme, the yarn may form a core layer of fibers with irreversible deformations.

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It is well known that the mechanical properties of the yarn depend on individual fibers deformations and occurrence contact forces interaction between them under twisting the yarn. In this case indicators of forming yarn depend on the differences of mechanical properties of fibers, i.e. length and diameter (linier density) and also spinning conditions. The mechanical behavior of the yarn under tension in the work of Platt [1] is represented as a set of stretched fibers have fixed geometrical position, defined by the angle of their inclination to the yarn axis relatively, and thus the applied tensile stress causes the fibers tension. Thus, at a maximum stress of fibers in general (yarn), each individual fiber exhibits an effective resistance to the current load. Moreover, the inelasticity of the fiber can be expressed as a result of diversity in the slope of stress-strain curve. At the same time, depending on their geometric configuration, the average stress is converted to local fiber bundle stress, and as the main indicator is the lateral pressure, which increases from zero at the outer surface of the yarn to the center. If there is no pressure, then the two adjacent fibers will easily slide relative to each other. However, if the pressure between them is very high, the full transfer of stress fibers will often break before they slip, and the axial stress at the end points vanishes.

The foregoing description of the behavior of the yarn under tension points to the possibility of development and improvement of representations of yarn mechanical models. The tensile properties of yarn and the effect of twist amount, twisting tension, stress distribution on the yarn structure have been discussed by many researchers [2-14]. In the initial stage of loading fibers oriented in the direction of the load. At the same time begin to be stretched in the presence of contact friction force, but fibers located at an angle to the direction of this force, partly stretched, slide and move together with the parallel oriented fibers. By applying twist the fibers start to settle down along helical lines, and a simultaneous decrease in the cross-section of yarn as a result of convergence of the fibers. When removing the axial force in the fibers remain irreversible deformation and, in some fibers – the residual stresses. This case leads to irregular distribution of stresses over the cross section and it is one of the reasons of product irregularity among its length. With this regard to above it is proposed the mechanical model of fiber deformation which, depending on their level of loading, subject to the elastic and elastic-plastic deformation of the law.

THEORETICAL APPROACH

While theoretical investigation of this research the following symbols were used as:

a– various values of the parameter,

b – length of slippage region, mm,

$c = cosa$,

E_f – Young's modulus for fibre (axial modulus of fibres), N/m^2,

E_{fr} – frictional force, N,

E_p – pulling force, N,

g – function associated with lateral compression of fiber,

G – specific stress, perpendicular to fibre axis, N/m^2,

h –length of one turn of twist, mm,

r_0 – radius of fibre investigated, mm,

r – distance of yarn element from center, mm,

$r^* = r/R$,

R – yarn radius, mm,

l – length of fibre path in one turn of twist, mm,

$l = h/cos\theta$,

$L_b = 2pr_0$,

$u = c/cos\theta$,

X – tensile specific stress of fibres in yarn, N/m^2,

a – yarn twist angle, deg,

θ – corresponding helical angle at radius r, deg,

e_y – yarn deformation,

e_f – fibre extension,

σ_1 – Poisson's ration for longitudinal deformation of fibre (axial Poisson's ratio),

σ_y – Poisson's ratio for yarn (lateral contraction ratio of yarn),

m – coefficient of friction between fibres.

The laws of deformation of textile yarn formed by fibers with different properties can be described by linear or nonlinear relationships between tension, elongation, and their time derivatives. The nature of these relationships, depending on the structural framework can be for the same yarn different. The qualitative aspect of the real deformation of the yarn, described such laws are generally satisfactory, but the quantitative ratio, defined by them sometimes differ significantly from reality. It can be to fix the laws so that they more accurately describe the deformation of the real yarn by introducing new constants in the mathematical expression of these laws. Thus, the structural irregularity heterogeneity of the structure of yarn caused by different types of fibers deformations and a large range of variation of their properties can also be described in sufficient strain state of the yarn and in quantitative terms. For this purpose, should be submitted to the actual yarn as a set of many elements, with the simplest laws of deformation, but with different constants in terms of these laws, choosing the appropriate distribution of such items.

Consider the case where the fibers in the axial loading and twisting of the yarn are deformed by the elastic or elastic-plastic laws of the Prandtl scheme. The axial stress in the fibers along the radius of the yarn can be represented by the equation [12].

$$X = E_f \varepsilon_y (\cos^2\theta - \sigma_y \sin^2\theta - 2\sigma_1 g) \ . \quad (1)$$

The angle θ is defined by following equation

$$\cos\theta = \frac{h}{\sqrt{h^2 + 2\pi^2 r^2}} \ . \quad (2)$$

The function $g = g(r)$ is associated with lateral compression of the fiber and is determined in accordance with [12] as follows:

$$g = \frac{1+\sigma_y}{1+2\sigma_1} \cdot \frac{c^2}{u^2}\left(1 - u^{1+2\sigma_1}\right) - \sigma_y \frac{1-u^{2\sigma_1 - 1}}{2\sigma_1 - 1} \ . \quad (3)$$

In [13] proposed another equation for calculating the function $g(r)$

$$g(r) = (\cos^2\theta - \sigma_y \sin^2\theta) \frac{\left(1 - \frac{r^2}{R^2}\right)\sin^2\alpha \cdot \cos^2\alpha}{2\left[\left(\frac{r}{R}\right)^2 \sin^2\alpha + \cos^2\alpha\right]} \ . \quad (4)$$

Numerical calculations made for the stress (1) using two types of functions $g(r)$, show their largest difference (10 %) for values $\sigma_1 = \sigma_y = 0{,}5$.

Figure 1 shows the curves of the reduced stress $x = X / E_f \sigma_y$ on the yarn radius for different twist angles α. Analysis of the curves in Figure 1 shows that the greatest value of the stress (tension) is reached in the fibers located in the central axis of the yarn. This indicates the possibility of developing plastic deformation in the central fiber at first and then by increasing strain – ε_y in the yarn it is formed the central zone, where deformation of the fibers occurs according to plastic law.

Denote by X_s limit stress value at which plastic deformation begins. Consider first the case of deformation of the fibers in the absence of lateral pressure, i.e., consider $g = 0$ In this case, the zone boundary $r = r_s$ plastic deformation of the fibers is determined by (1), where it should be assumed $X = X_s$ at $\theta = \theta_s$, which gives:

$$X_s = E_s \varepsilon_y \left(\cos^2\theta_s - \sigma_y \sin^2\theta_s\right) \ . \quad (5)$$

Solving (5) with respect to θ_s we find

$$\theta_s = arctg\sqrt{\frac{\varepsilon_y - \varepsilon_s}{\varepsilon_y \sigma_y + \varepsilon_s}} \ . \quad (6)$$

Valid values of θ_s are determined from condition

$$\varepsilon_y \geq \varepsilon_s \ . \quad (7)$$

Inequality (7) is a condition for the axial deformation – ε_y of the yarn, under which the fiber goes into plastic deformation. The boundary of this zone $r = r_s$ is determined from the equation (2):

$$\frac{r_s}{R} = \frac{tg\theta_s}{tg\alpha} \ . \quad (8)$$

From condition $r_s / R \leq 1$ should be $tg\theta_s < tg\alpha$ or using (6) we have

$$\varepsilon_y \leq \varepsilon_y^x = \frac{\varepsilon_s}{\cos^2\alpha - \sigma_y \sin^2\alpha} \ . \quad (9)$$

Thus, if the axial deformation ε_y satisfies the condition $\varepsilon_s \leq \varepsilon_y \leq \varepsilon_y^x$, the section of the yarn produced two zones: the elastic $r_s \leq r \leq R$ and plastic $0 \leq r \leq r_s$. If there is condition $\varepsilon_y \geq \varepsilon_y^x$ then

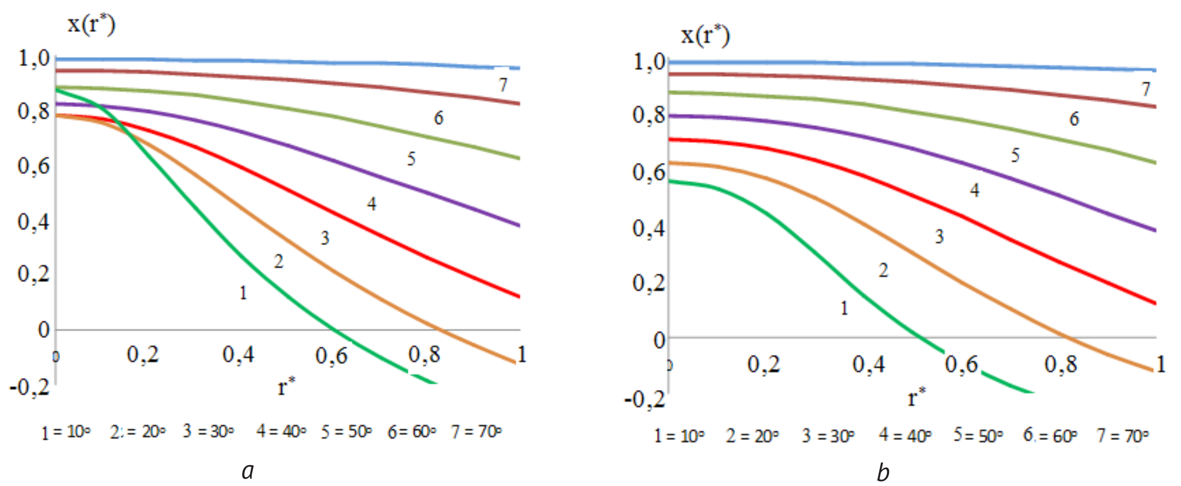

Figure 1 – The curves distribution of tensile stress of fibers (given) $x = X / E_f\varepsilon_y$ *at radii* $r^* = r / R$ *for various values of twist angles* α *and* $\sigma_1 = 0.5$, $\sigma_y = 0.5$: *a – by Hearle's equation [12, equation 4.45]; b – by proposed equation (4)*

the zone of elastic deformation of fibers will be absent, i.e., all fibers in the yarn will be in a state of plastic deformation.

In the zone of plastic deformation the connection between stress X and deformation of the fiber is determined by the Prandtl scheme:

$$X = (E_f - E_{1f})\varepsilon_s + E_{1f}\varepsilon_y\left(\cos^2\theta - \sigma_y \sin^2\theta\right) \quad \text{at } 0 \le r \le r_s. \qquad (10)$$

In the area of elastic deformation of the fibers we have

$$X = E_f\varepsilon_y\left(\cos^2\theta - \sigma_y \sin^2\theta\right) \quad \text{at} \quad r_s \le r \le R. \quad (11)$$

In the presence of lateral pressure G stresses in each zone is determined by the equations

$$X = (E_f - E_{1f})\varepsilon_s + \varepsilon_y E_{1f}\left(\cos^2\theta - \sigma_y \sin^2\theta - 2\sigma_1 g(r)\right) \quad \text{at} \quad 0 \le r \le r_s, \qquad (12)$$

$$X = \varepsilon_y E_f\left(\cos^2\theta - \sigma_y \sin^2\theta - 2\sigma_1 g(r)\right) \quad \text{at} \quad 0 \le r \le r_s \quad (13)$$

and the border zone of two types of deformation is determined from the equation:

$$X_s = E_f\varepsilon_y\left[\cos^2\theta_s - \sigma_y \sin^2\theta_s - 2\sigma_1 g(r_s)\right], \quad (14)$$

where

$$g(r_s) = \frac{1+\sigma_y}{1+2\sigma_1}\cdot\frac{c^2}{u_s^2}\left(1 - u_s^{1+2\sigma_1}\right) - \sigma_y\frac{1-u_s^{2\sigma_1-1}}{2\sigma_1-1}, \quad u_s = \frac{\cos\alpha}{\cos\theta_s}. \quad (15)$$

Thus, under certain values of X_s, E_1, E_y, σ_y, σ_1 and α the expression (9) is a transcendental equation for determining the angle θ_s, knowing that from (2) further it can be find unknown radius r_s. If we use the expression (4) for determining angle θ_s we obtain the equation:

$$X_s = E_f\varepsilon_y\left(\cos^2\theta_s - \sigma_y \sin^2\theta_s\right)\left[1 - \sigma_1\left(\cos^2\theta_s - \cos^2\alpha\right)\right]. \quad (16)$$

Solving this equation for $\cos\theta_s$

$$\cos\theta_s = \sqrt{\frac{1}{2}\left[a + b - \sqrt{(a-b)^2 - 4C/\varepsilon_p}\right]}, \quad (17)$$

where

$$a = \frac{\sigma_y}{1+\sigma_y}, \quad b = \frac{1+\sigma_1\cos^2\alpha}{\sigma_1}, \quad C = \frac{1}{\sigma_1(1+\sigma_y)}, \quad \varepsilon_p = \frac{\varepsilon_y}{\varepsilon_s}. \quad (18)$$

The condition of the existence of a real root, should:

$$\varepsilon_y \le \varepsilon_y^x = \frac{\varepsilon_s}{\cos^2\alpha - \sigma_y \sin^2\alpha}\ , \qquad (19)$$

since $\theta_s \le \alpha$ $(\cos\theta_s \ge \cos\alpha)$ $(0 \le \alpha \le 90^0)$ then from (17) we have

$$\frac{1}{2}\left[a+b-\sqrt{(a-b)^2-4C/\varepsilon_p^*}\right] \ge \cos^2\alpha\ . \qquad (20)$$

From this inequality we establish: $\varepsilon_y \le \varepsilon^*_y$ that is, the presence of lateral pressure does not affect the amount of deformation ε^*_y, which determines the limiting value of the deformation of yarn in which all the fibers pass into a state of plastic deformation.

Figure 2 shows the curves of the border zone r_s / R between the elastic and plastic deformation of the fibers from the relationship to $\varepsilon_p = \varepsilon_y / \varepsilon_s$ for $\sigma_1 = 0.1$, $\sigma_y = 0.5$ and different values of the twist angle.

Analysis of the graphs shows that with increasing twist angle the fibers in the yarn cross-sections is less stretched and therefore plastic deformation occurs at higher values of the axial deformation ε_y of the yarn. For example, at $\alpha = 20^0$ all the fibers in the yarn will be in a state of plastic deformation, if the axial deformation of the yarn has a value $\varepsilon_y > 1.15\varepsilon_s$ and if the twist angle is equal to $\alpha = 50^0$ then this value of deformation will be greater than $2.8\varepsilon_s$.

Figure 3 shows the curves of axial stress distribution $x = X / E_{1f}\varepsilon_s$ from the radius for two values of the twist angle α and different attitudes and values $k = E_{1f} / E_f$ and $\varepsilon_y / \varepsilon_s$. In the calculations taken: $\sigma_1 = 0.1$, $\sigma_y = 0.5$.

In the figure 3 the curves 1 are corresponding to the elastic deformation under $\varepsilon_y < \varepsilon_s$, condition while the curves 5 are deformed by plastic law.

Reduced coefficient of hardening - k can lead to a significant decrease in axial stress fibers in the zone of plastic deformation. This effect is most clearly observed for large values of the twist angle α. For example, at twist angle $\alpha = 20^0$ the stress of fibers, located on the axis of the yarn is reduced by 1,4 times. When twist angle $\alpha = 50^0$ the stress in this fiber is reduced 2,82 times.

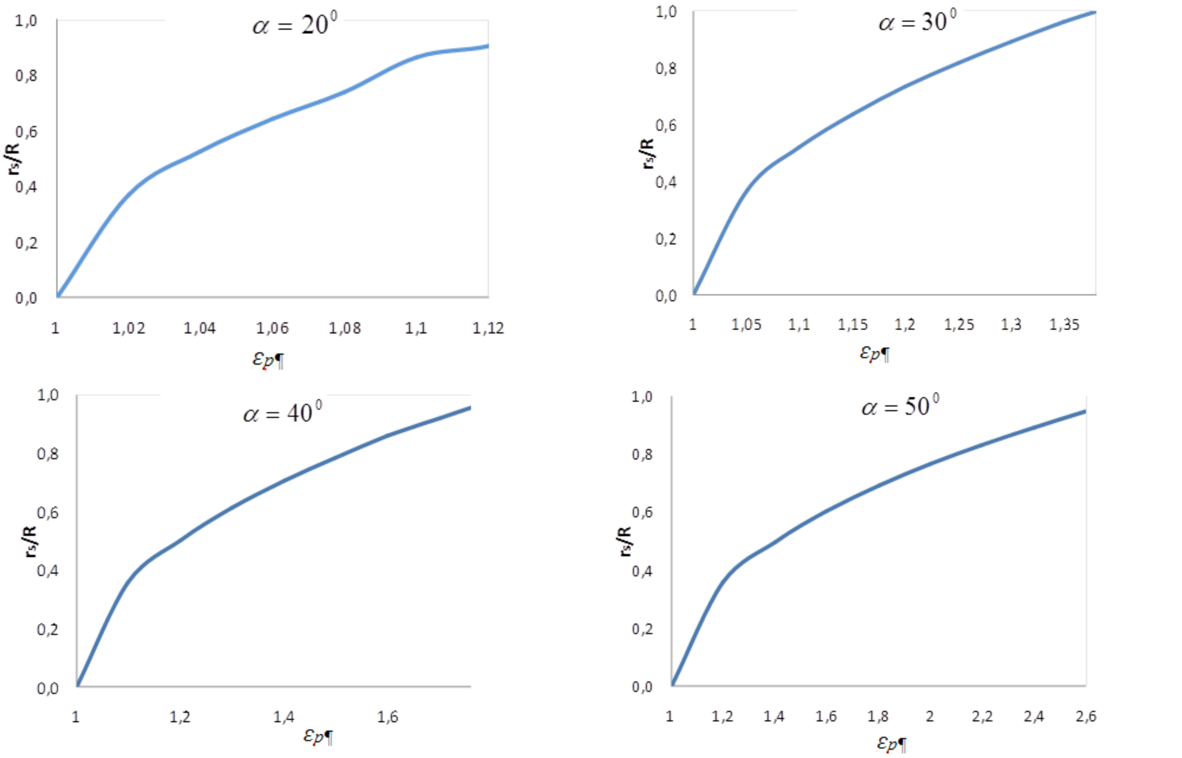

Figure 2 – Depending on the border zone of deformation r_s / R on the ratio $\varepsilon_p = \varepsilon_y / \varepsilon_s$ for different values of the angle α

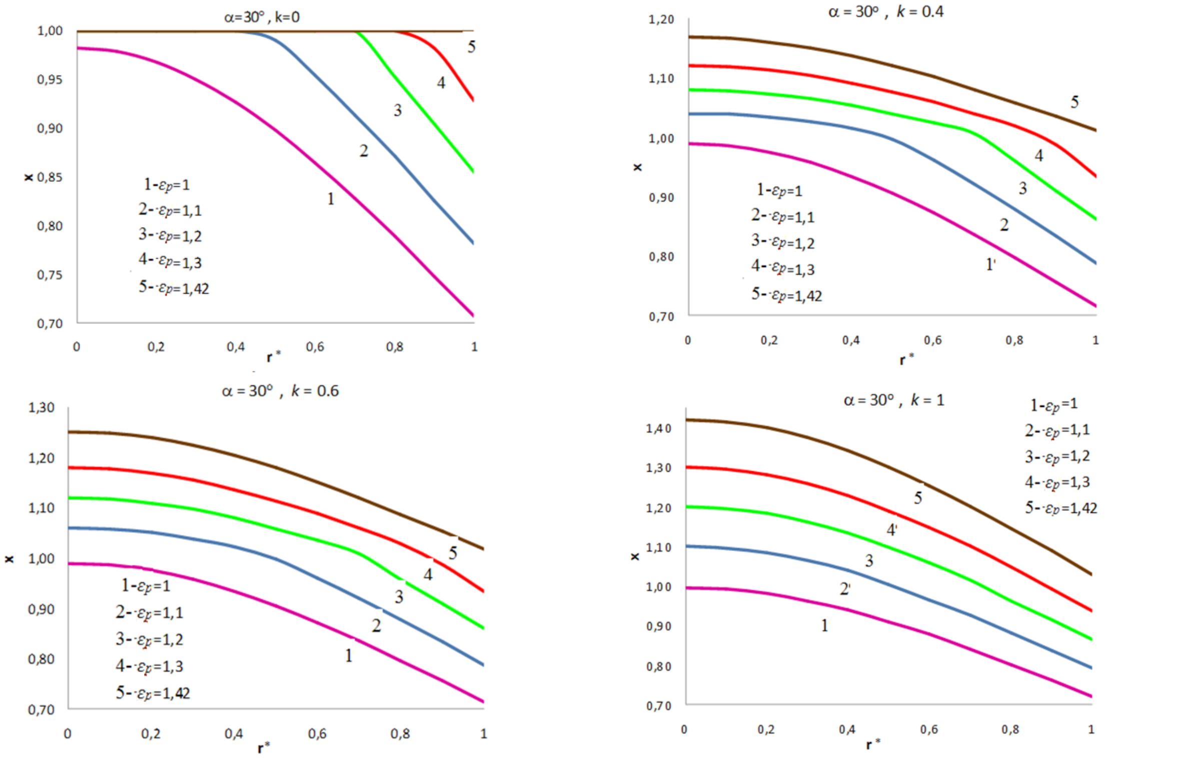

Figure 3 – The curves distribution of tensile stress of fibers (given) $x = X / E_f \varepsilon_y$ *at radii* $r^* = r / R$ *at twist angle* $\alpha = 30^0$ *for various values of* $k = E_{1f} / E_f$ *and* $\varepsilon_p = \varepsilon_y / \varepsilon_s$

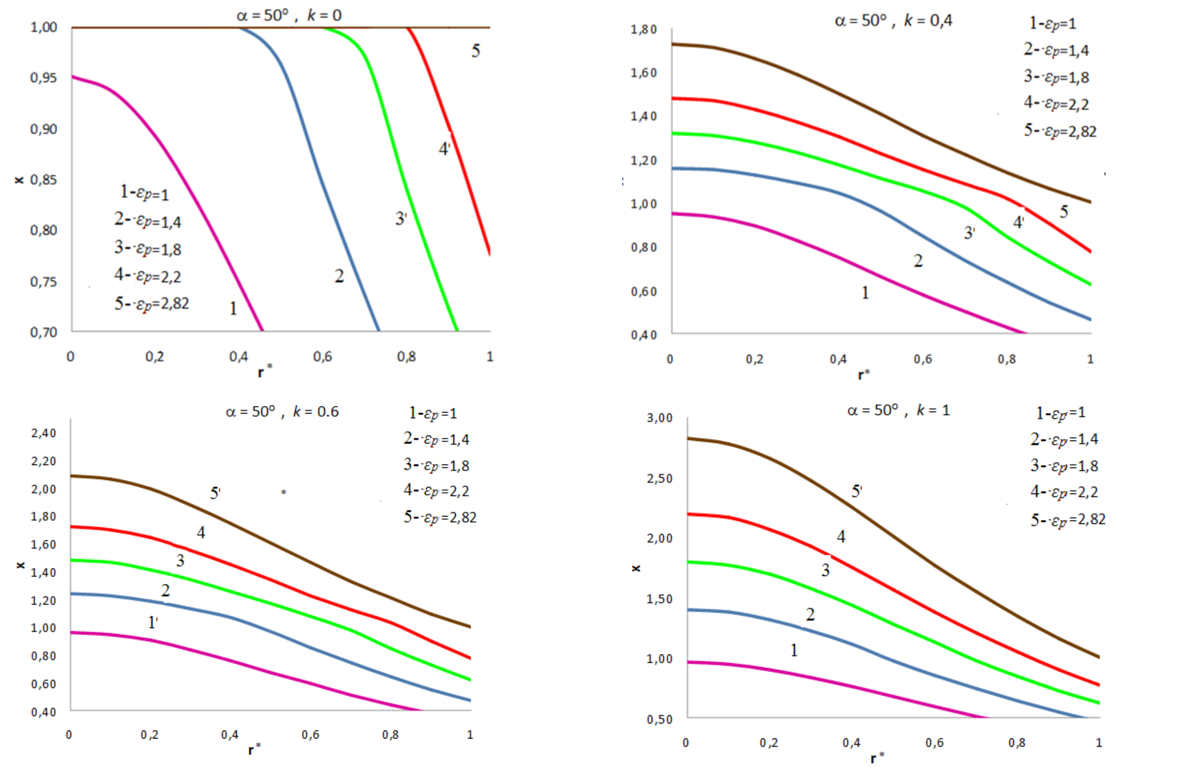

Figure 4 –The curves distribution of tensile stress of fibers (given) $x = X / E_f \varepsilon_y$ *at radii* $r^* = r / R$ *at twist angle* $\alpha = 50^0$ *for various values of* $k = E_{1f} / E_f$ *and* $\varepsilon_p = \varepsilon_y / \varepsilon_s$

CONCLUSION

During analytical investigation the statement of deformed yarn was developed the theoretical model of deformation of the fibers, depending on the values of axial force and the twist angle of the yarn. For describing the development of plastic deformation zone of the fibers proposed to use the Prandtl scheme, where the link between the deformation of fibers and the axial stress is represented by two straight lines (bilinear diagram). It is found that when the axial strain value is $\varepsilon_y = \varepsilon^x_y = \varepsilon_s / (\cos^2\alpha - \sigma_y \sin^2\alpha)$ all fibers of the yarn will be in a state of plastic deformation. In the absence of hardening ($k = 0$) the stress in the plastic zone when the twist angle is $10^0 < \alpha < 50^0$ can be reduced by 1,5–3 times. Thus, in the presence of deformed fibers in the yarn by elastically-plastic law according to the Prandtl scheme, there is can be developed the central layer of fibers in the yarn with irreversible deformations.

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